Conclusions

The critical length of the afterbody of a cone-cylinder probe on a slender nosed (12° half angle) vehicle has been investigated experimentally at Mach numbers from 1.4 to 4.0 and angles of attack up to 12°. In transitional flow with adiabatic wall temperature and at freestream Reynolds numbers from 7.0×10^5 to 8.0×10^6/ft, the critical length increases with increasing Mach number and angle of attack and decreases with increasing Reynolds number. In turbulent flow, the critical length is relatively small and apparently independent of Mach number, Reynolds number, and angle of attack.

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Measurement of Mean Particle Size in a Gas-Particle Flow

R. A. Dobbins*

Brown University, Providence, R. I.

Nomenclature

c_n = particle number concentration
D = particle diameter
D_∞ = maximum particle size present
F/F_0 = optical transmission
$K(D,m)$ = extinction coefficient or scattering coefficient for dielectric particles
l = optical path length
m = refractive index of particle relative to surrounding medium
$N_r(D)$ = particle size distribution function

THE use of metallic additives in solid propellants results in the formation of particulate products of combustion, the sizes of which influence the performance of the rocket motor. Losses resulting in reduced thrust will occur if the particle size is not small enough to assure thermal and dynamic equilibrium between particulate and gaseous phases The importance of techniques of measuring particle size therefore is apparent. Probably the most accurate method of measuring particle size distribution is to obtain a representative sample for observation by appropriate methods of microscopy. The work reported by Sehgal[1] appears to be an excellent example of the application of this type of technique. Although the most accurate means of measurement is preferred on many occasions, it is natural to inquire as to whether other techniques are available which possess advantages of greater convenience or more universal applicability. The scattering properties of the particles appear to provide a basis for a technique that fulfills this description.

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* Assistant Professor of Engineering. Member AIAA.

The advantages of a technique based on scattering properties are, as noted by van de Hulst,[2] the rapidity of the evaluation and its applicability when the particles are inaccessible, i.e., when they cannot be sampled properly. A starting point for an investigation on the use of scattering properties in the simplest possible manner is afforded by considering the optical transmission of the gas-particle fluid.

The transmission law applicable for a polydispersion of spherical particles is

$$\frac{F}{F_0} = \exp\left[-\frac{\pi}{4} c_n l \int_0^{D_\infty} K(D,m) N_r(D) D^2 dD\right] \tag{1}$$

The particle size distribution function $N_r(D)$ is defined such that

$$\int_{D_1}^{D_2} N_r(D) dD = P[D_1 < D < D_2] \tag{2}$$

where $P[D_1 < D < D_2]$ represents the relative probability of occurrence of particles larger than D_1 and smaller than D_2. The extinction coefficient is determined from electromagnetic theory as described by van de Hulst.[2]

The transmission law can be written in a more useful form by introducing certain integrated quantities. The mean extinction coefficient $\bar{K}$ and the volume-to-surface mean diameter are defined as

$$\bar{K} \equiv \frac{\int_0^{D_\infty} K(D,m) N_r(D) D^2 dD}{\int_0^{D_\infty} N_r(D) D^2 dD} \tag{3}$$

$$D_{32} = \frac{\int_0^{D_\infty} N_r(D) D^3 dD}{\int_0^{D_\infty} N_r(D) D^2 dD} \tag{4}$$

and the particle volume concentration c_v is introduced:

$$c_v = \frac{\pi}{6} c_n \int_0^{D_\infty} N_r(D) D^3 dD \tag{5}$$

The transmission law now can be expressed as

$$\frac{F}{F_0} = \exp\left(-\frac{3}{2}\frac{\bar{K} c_v l}{D_{32}}\right) \tag{6}$$

Although the determination of a complete distribution function from an optical transmission test generally is not possible, Eq. (6) suggests that a determination of the mean diameter represented by D_{32} may be feasible.

For the purpose of exploring this possibility, the mean scattering coefficient $\bar{K}$ was calculated over a wide range of D_{32} initially for rectangular and parabolic distributions of dielectric particles, the refractive index of which is near unity, $(m - 1) \ll 1$. The results show that the largest discrepancy between the mean scattering coefficients yielded by these two radically different distribution functions for given D_{32} was only 3% and that usually the two coefficients were more nearly coincident. The mean scattering coefficient for skewed distribution functions possessing a finite maximum diameter D_∞ now is being investigated. The results obtained thus far indicate that the $\bar{K}$ for these functions, which are empirically representative of particle size distributions, is very close to that of the parabolic distribution possessing the same value of D_{32}. A study of mean scattering coefficient for polydispersions of dielectric particles of finite $(m - 1)$ is now in progress. The presently available results indicate that the $\bar{K}$ is primarily a function of D_{32} independent of the shape of the distribution function, or, in alternate form,

$$D_{32} = f(\bar{K}/D_{32}) \tag{7}$$

Figure 1 may be regarded as expressing this functional relationship in appropriate nondimensional form for parabolic and rectangular distribution functions of particles of small $(m - 1)$. Thus, an optical transmission test on a gas-

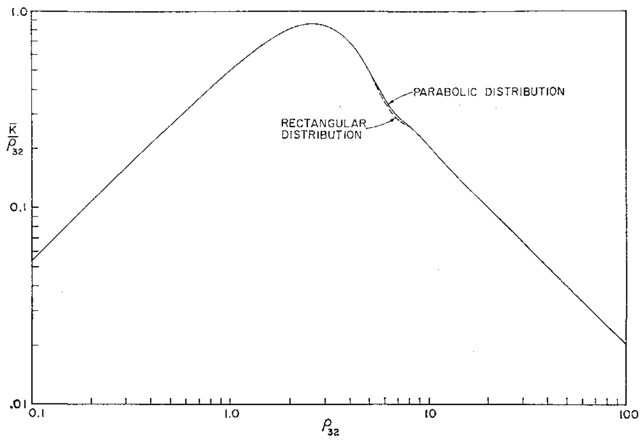

Fig. 1 Variation of scattering cross section $\bar{K}/\rho_{32}$ for parabolic and rectangular distributions of dielectric particles as a function of $\rho_{32} = 2(m - 1)(\pi D_{32}/\lambda)$, where λ equals the wavelength of incident light [applicable for $(m - 1) \ll 1$, $(\pi D_{32}/\lambda) \gg 1$].

particle fluid will permit a determination of D_{32} through Eqs. (6) and (7) if the particle volume fraction c_v is known.

The particle volume fraction can be found in terms of the particle mass fraction c_m, assuming one-dimensional flow, by dividing the continuity equation for the particulate phase by the continuity equation for the gaseous phase to yield

$$c_m = \frac{\pi}{6}\frac{\rho_p}{\rho_g} c_n \int_0^{D_\infty} \frac{V(D)}{V_g} N_r(D)D^3dD \qquad (8)$$

The quantities ρ_p and V_p designate density and velocity, respectively, of particulate phase, whereas ρ_g and V_g represent the corresponding quantities for the gaseous phase. The particle mass fraction is known from the chemical composition of the propellants if complete combustion occurs. The particle velocity at any given nozzle station, a function of diameter, generally will depend on the entire history of the particle trajectory and can be calculated by methods described by various investigators.[3-5] Recognizing the velocity ratio contained in (8) as the dynamic lag coefficient K_l, it is convenient to define a mean value of this parameter as

$$\bar{K}_l \equiv \frac{\int_0^{D_\infty} K_l(D)N_r(D)D^3dD}{\int_0^{D_\infty} N_r(D)D^3dD} \qquad (9)$$

where $K_l(D) = V_p(D)/V_g$.

Combining (8) and (9) gives the following expression for the particle volume fraction:

$$c_l = (\rho_g/\rho_p)(c_m/\bar{K}_l) \qquad (10)$$

The mean dynamic lag coefficient does not lend itself readily to experimental determination, and a calculated value will depend on a knowledge of the size distribution function that is not known. This difficulty can be circumvented by noting that dynamic and thermal lag should be related directly, since momentum and heat transfer processes from gas to particle are related closely. The following relation due to Kliegel[3] is found by examining the equations of motion of gaseous and particulate phases and taking the Stokes drag law and the limiting value of the Nusselt number equal to 2:

$$L_l = \frac{1}{1 + 3Pr(c_p/c_g)[(1 - K_l)/K_l]} \qquad (11)$$

where $L_l \equiv (T_0 - T_p)/(T_0 - T_g)$, and Pr is the Prandtl number of the gas, c_p/c_g is the ratio of specific heat of particulate to gaseous phase, T_0 is the combustion chamber temperature, and T_p and T_g are the local particle and gas temperatures, respectively. The product $(c_p/c_g)Pr$ is close to unity, and Eq. (11) simplifies to

$$L_l = K_l/(3 - 2K_l) \qquad (12)$$

The validity of Eq. (12) can be checked with the results of specific particle trajectory calculations (Fig. 2). The calculations of Bailey et al.[4] show moderately good agreement with Eq. (12), even though L_l and K_l apparently were not assumed to be constant, as required for the validity of (11) and (12). A simple linear relation that closely approximates Eq. (12) for values of $L_l > 0.5$ will be used in present work:

$$L_l = 2K_l - 1.0 \qquad (13)$$

Using this relationship, the mean dynamic lag parameter $\bar{K}_l$ is related to the mean thermal lag parameter as follows:

$$\bar{K}_l = 1/2(\bar{L}_l + 1) \qquad (14)$$

where

$$\bar{L} \equiv \frac{\int_0^{D_\infty} L_l N_r(D)D^3dD}{\int_0^{D_\infty} N_r(D)D^3dD} \qquad (15)$$

Optical methods have been used by Carlson[6] and others to measure particle temperature, and it is through such techniques that one may hope to measure $\bar{L}_l$ and hence $\bar{K}_l$. It is not readily apparent that the mean particle temperature measured by optical methods is the correct temperature to use in calculating the thermal lag coefficient as defined in the foregoing. This must be checked with the aid of calculations of the particle trajectory for given size distributions and hence temperature histories. If this approach is not possible, then it is necessary to restrict the technique to cases where it is possible, a posteriori, to verify that particles several times larger than the measured D_{32} would be in equilibrium with the gas phase. This method of measuring particle size is most accurate and can be applied most directly when the particles are in dynamic equilibrium and hence $\bar{K}_l$ is unity.

Data given by Carlson,[6] who performed optical transmission tests on a gas-particle flow incident to measuring gas and particle temperatures when there existed a departure from thermal equilibrium between the gaseous and particulate phases, offer the opportunity to test the theory of the foregoing method of measuring mean particle size. From a detailed particle size analysis supplied by Carlson,[7] one calculates a volume-to-surface mean diameter of 3.1 μ. The experimentally determined optical depth and other data supplied by Carlson, when used as inputs to the foregoing theory, indicate a value of D_{32} equal to 2.9 μ. The close agreement between the theoretical and experimental values of D_{32} is regarded as fortuitous in view of the various uncer-

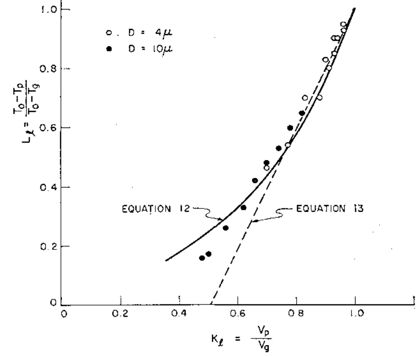

Fig. 2 Dependence of thermal lag parameter on dynamic lag parameter from results of trajectory calculations reported by Bailey et al.[4]

tainties to which the experiment is subject. However, these results are considered highly encouraging and indicate that the further development of the method is a worthy pursuit.

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Optimum Toroidal Pressure Vessel Filament Wound along Geodesic Lines

James D. Marketos*

Goodyear Aerospace Corporation, Akron, Ohio

Nomenclature

x,y	=	Cartesian coordinates
y',y''	=	first and second derivative of y with respect to x
σ	=	principal stress, lb/in.
ρ	=	principal radius of curvature
R	=	radial distance of any point on toroid to axis of rotational symmetry
ω	=	angle $\tan^{-1}(dy/dx)$
β	=	variable wrap angle, i.e., angle of the filament with the meridian at any point
v	=	quantity $(1 + y'^2)^{1/2}/y'$ or $1/\sin\omega$
n	=	factor of safety
N	=	total number of turns of one set of filaments required for construction of toroid
p	=	gage pressure of inflation gas
T	=	strength of filament
S_b	=	stress in direction of either set of filaments at a point, lb/in.
A,C	=	constant quantities
$f(\beta),F(\beta)$	=	functions of wrap angle

Subscripts

i	=	toroid inner equator
o	=	toroid outer equator
m	=	toroid parallel circle passing through origin of coordinates
1	=	meridional direction
2	=	circumferential direction

Discussion

Let $a - a$ be the axis of rotational symmetry of a toroidal surface, the meridian of which is shown only partly in Fig. 1a by the curve OM. In Fig. 1b are shown two filaments passing through a point M of the toroid; these filaments are arranged symmetrically with respect to the meridian at that point.

Considering as a free body the circular band generated by the arc OM rotating 360° about the axis of rotational symmetry and writing equilibrium of forces along this axis gives

$$2\pi(R_m + x)\sigma_1 \sin\omega = \pi[(R_m + x)^2 - R_m{}^2]p \qquad (1)$$

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* Engineering Specialist.

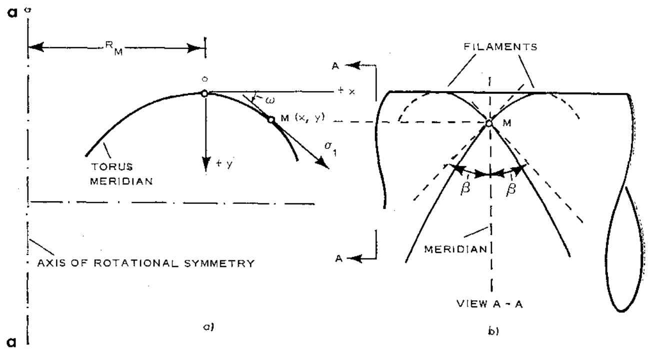

Fig. 1 a) Toroid meridional section; b) coordinates and wrap angle.

Solving Eq. (1) for σ_1 yields

$$\sigma_1 = \frac{x(2R_m + x)p}{2(R_m + x)\sin\omega} \qquad (2)$$

Substituting Eq. (2) into

$$(\sigma_1/\rho_1) + (\sigma_2/\rho_2) = p \qquad (3)$$

(see Ref. 1, p. 160), noting that

$$\rho_1 = (1 + y'^2)^{3/2}/y'' \qquad (4)$$

$$\rho_2 = (R_m + x)/\sin\omega \qquad (5)$$

and solving the resulting equation for σ_2 yields

$$\sigma_2 = \frac{R_m + x}{\sin\omega}\left[1 - \frac{x(2R_m + x)y''}{2(R_m + x)(1 + y'^2)^{3/2}\sin\omega}\right] \qquad (6)$$

Noting that

$$\sin\omega = \frac{\tan\omega}{(1 + \tan^2\omega)^{1/2}} = \frac{y'}{(1 + y'^2)^{1/2}} \qquad (7)$$

Eqs. (2) and (6) become, respectively,

$$\sigma_1 = \frac{x(2R_m + x)(1 + y'^2)^{1/2}p}{2(R_m + x)y'} \qquad (8)$$

$$\sigma_2 = \frac{(R_m + x)(1 + y'^2)^{1/2}}{y'}\left[1 - \frac{x(2R_m + x)y''}{2(R_m + x)y'(1 + y'^2)}\right]p \qquad (9)$$

Let β be the wrap angle, i.e., the variable angle between the meridional line at a point and each of the two filaments passing through this point (see Fig. 1b), N be the total number of turns of the one set of filaments required for the construction of the toroid, and T be the strength of the filaments in pounds. The number of filaments which cross the unit length of the parallel circle passing through the point M is $N/2\pi(R_m + x)$; and the strength S_b in pounds per inch of each set of filaments is

$$S_b = NT/2\pi(R_m + x)\cos\beta \qquad (10)$$

Let P be the force carried by the $N/2\pi(R_m + x)$ filaments of one set in the direction of the filaments. From Fig. 2 it is clear that

$$P = S_b \cos\beta \qquad (11)$$

The component of P along the meridional direction is $P\cos\beta$ or $S_b\cos^2\beta$. Hence, for the two sets of filaments this component is $2S_b\cos^2\beta$. If n is the desired factor of safety, the force $2S_b\cos^2\beta$ corresponds to an actually applied stress $2S_b\cos^2\beta/n$, which acts along the meridional direction; therefore it is equal to σ_1. Hence,

$$n\sigma_1 = 2S_b\cos^2\beta \qquad (12)$$

Similarly, the component of P along the parallel circle is